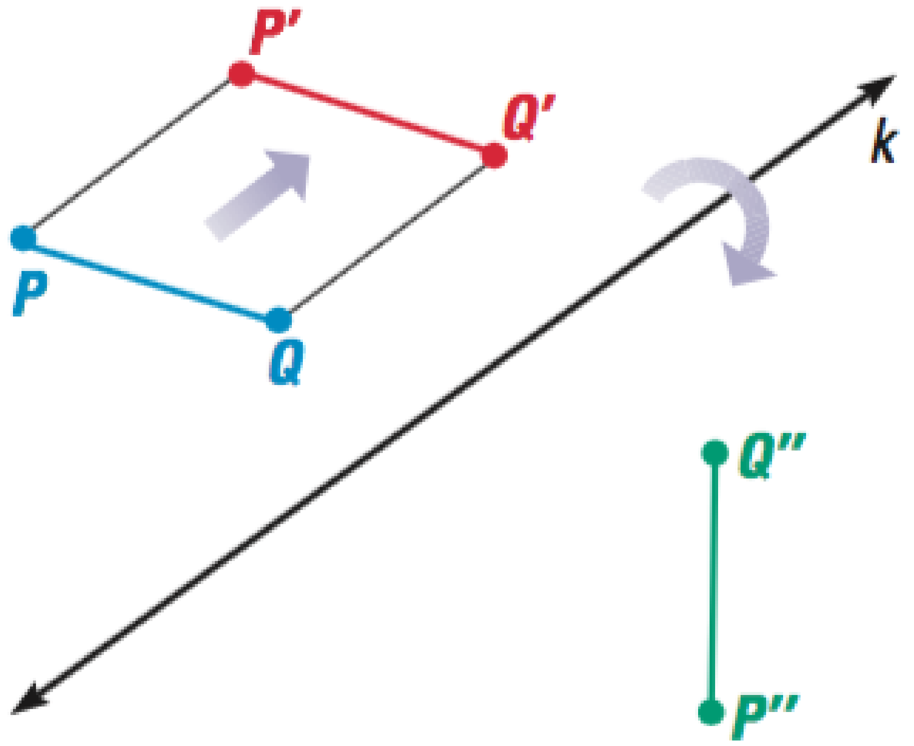
Chapter 7
Transformations

Section 5
Glide Reflections and Compositions

GOAL 1: Using Glide Reflections

A translation, or glide, and a reflection can be performed one after the other to produce a transformation known as a *glide reflection*. A **glide reflection** is a transformation in which every point P is mapped onto a point P'' by the following steps:



1. A translation maps P onto P'.
2. A reflection in a line k parallel to the direction of the translation maps P' onto P''.

As long as the line of reflection is parallel to the direction of the translation, it does not matter whether you glide first and then reflect, or reflect first and then glide.

Example 1: Finding the Image of a Glide Reflection

Use the information below to sketch the image of $\triangle ABC$ after a glide reflection.

$A(-1, -3)$, $B(-4, -1)$, $C(-6, -4)$

Translation: $(x, y) \rightarrow (x + 10, y)$

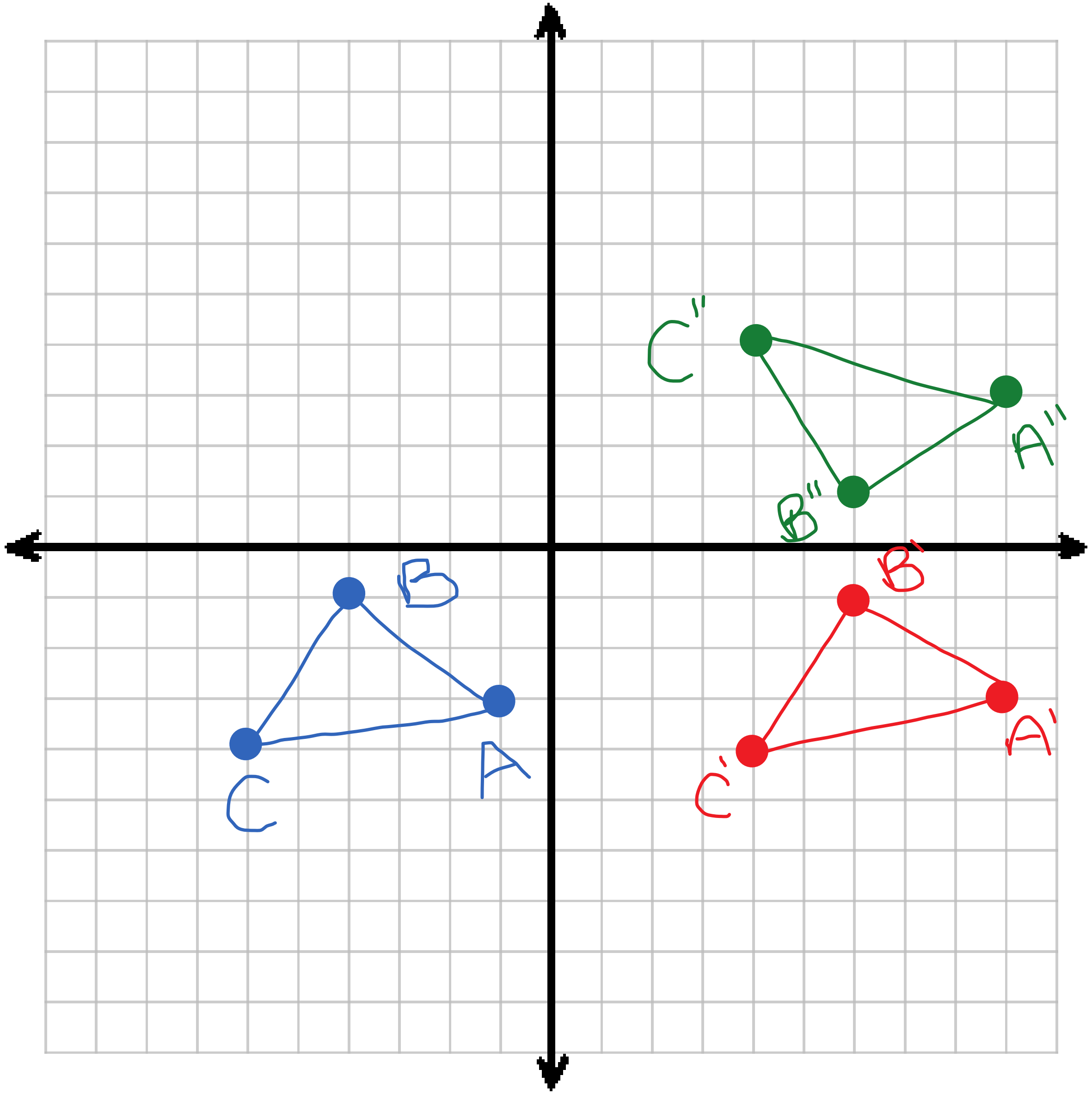
Reflection: in the x-axis

right 10, up 0

$A'(9,-3)$ $B'(6,-1)$ $C'(4,-4)$

Keep x, change y

$A''(9,3)$ $B''(6,1)$ $C''(4,4)$



GOAL 2: Using Compositions

When two or more transformations are combined to produce a single transformation, the result is called a **composition** of the transformations.

THEOREM

THEOREM 7.6 ***Composition Theorem***

The composition of two (or more) isometries is an isometry.

Because a glide reflection is a composition of a translation and a reflection, this theorem implies that glide reflections are isometries. In a glide reflection, the order in which the transformations are performed does not affect the final image. For other compositions of transformations, the order may affect the final image.

Example 2: Finding the Image of a Composition

Sketch the image of PQ after a composition of the given rotation and reflection.

$P(2, -2)$, $Q(3, -4)$

Rotation: 90° counterclockwise about the origin

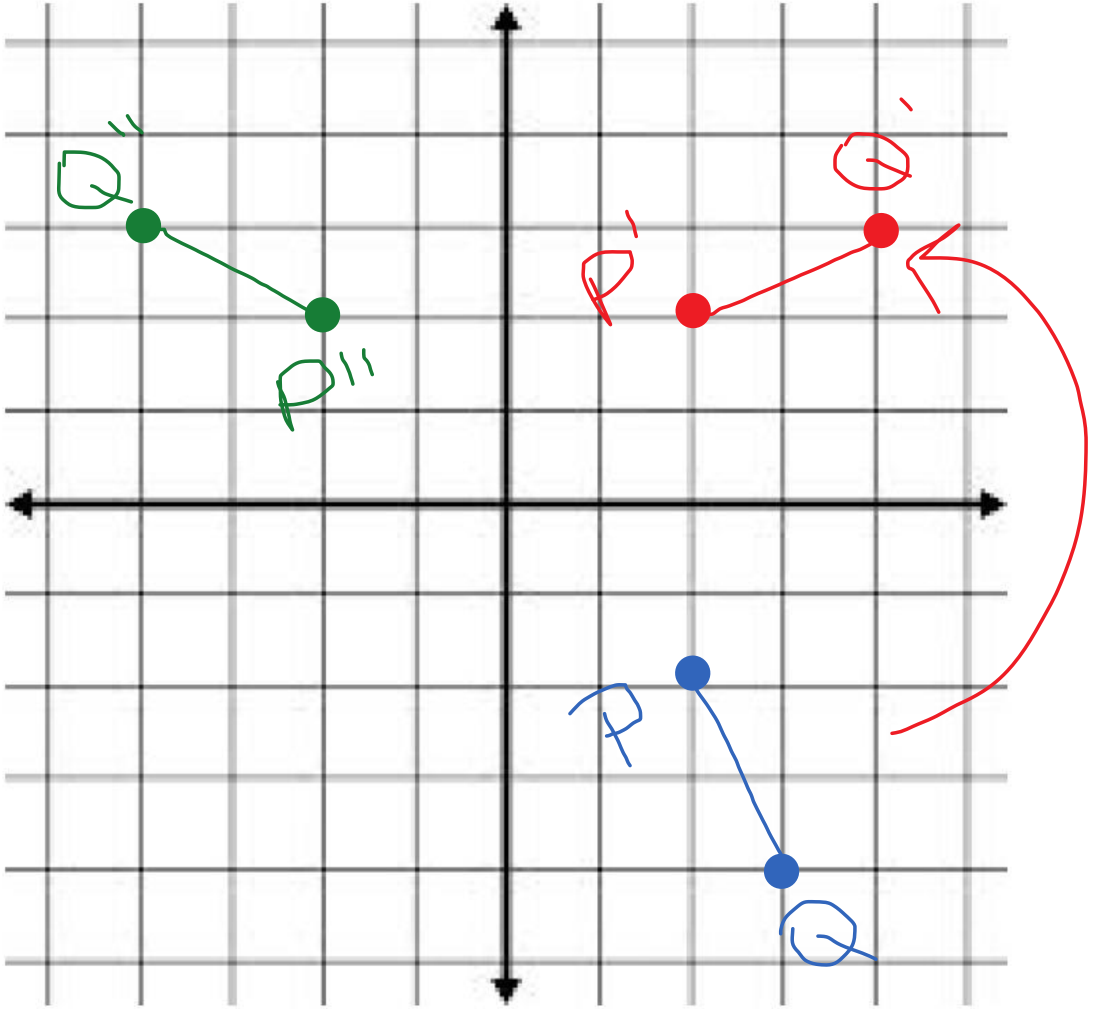
Reflection: in the y-axis

switch x+y, change sign of "new" x

P'(+2,2) Q'(+4,3)

keep y, change x

P''(-2,2) Q''(-4,3)



Example 3: Comparing Orders of Compositions

Repeat Example 2, but switch the order of the composition by performing the reflection first and the rotation second. What do you notice?

ORDER MATTERS

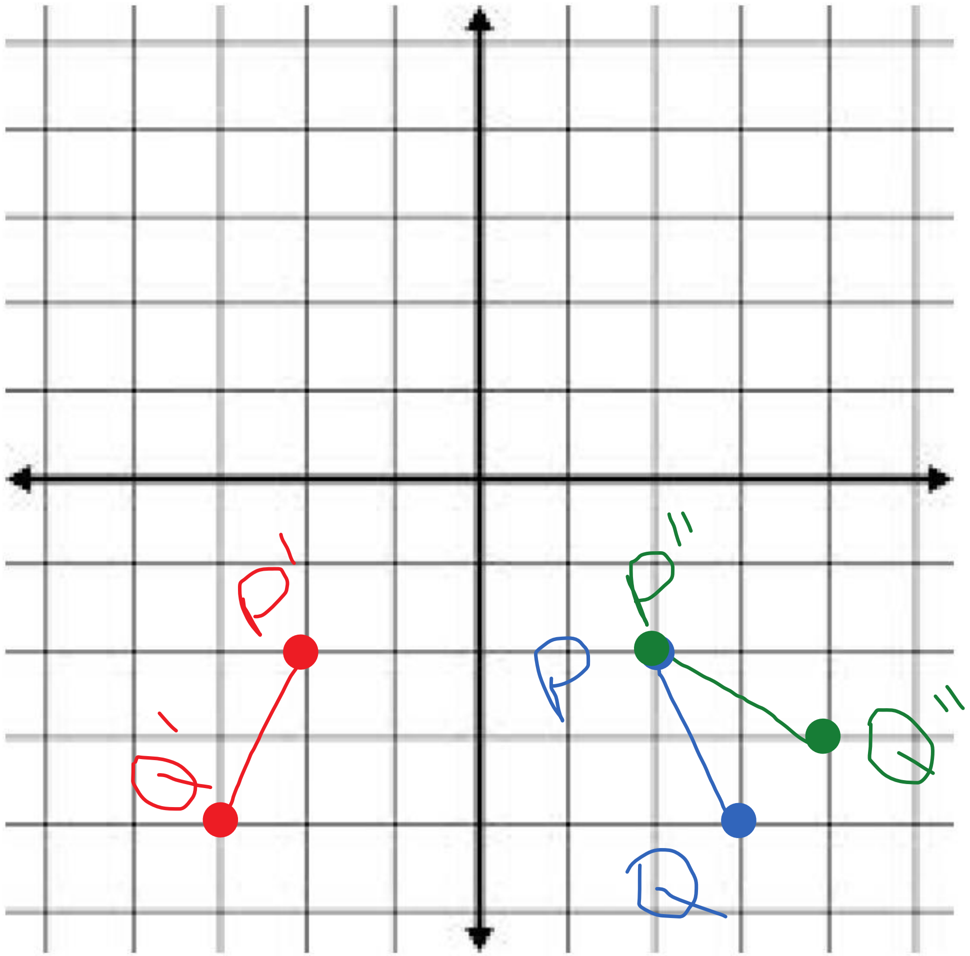
$P(2, -2)$, $Q(3, -4)$

Reflection: in the y-axis

Rotation: 90° counterclockwise about the origin

$P'(-2,-2)$ $Q'(-3,-4)$

$P''(+2,-2)$ $Q''(+4,-3)$



Example 4: Describing a Composition

*vertical lines → x = #; horizontal lines → y = #

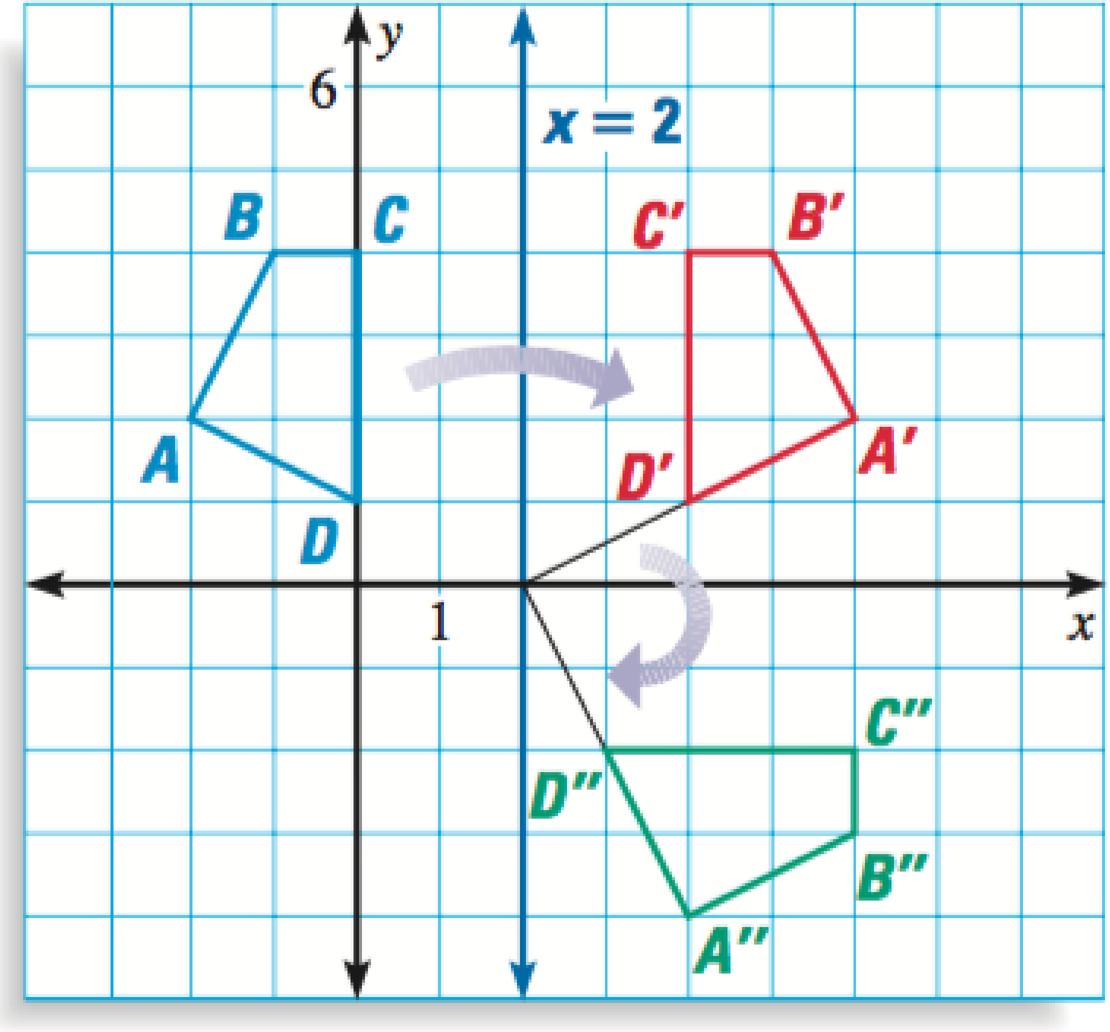
Describe the composition of the transformations in the diagram.

ABCD → A’B’C’D’

→ reflection across x = 2

A’B’C’D’ → A”B”C”D”

→ rotation 90* clockwise



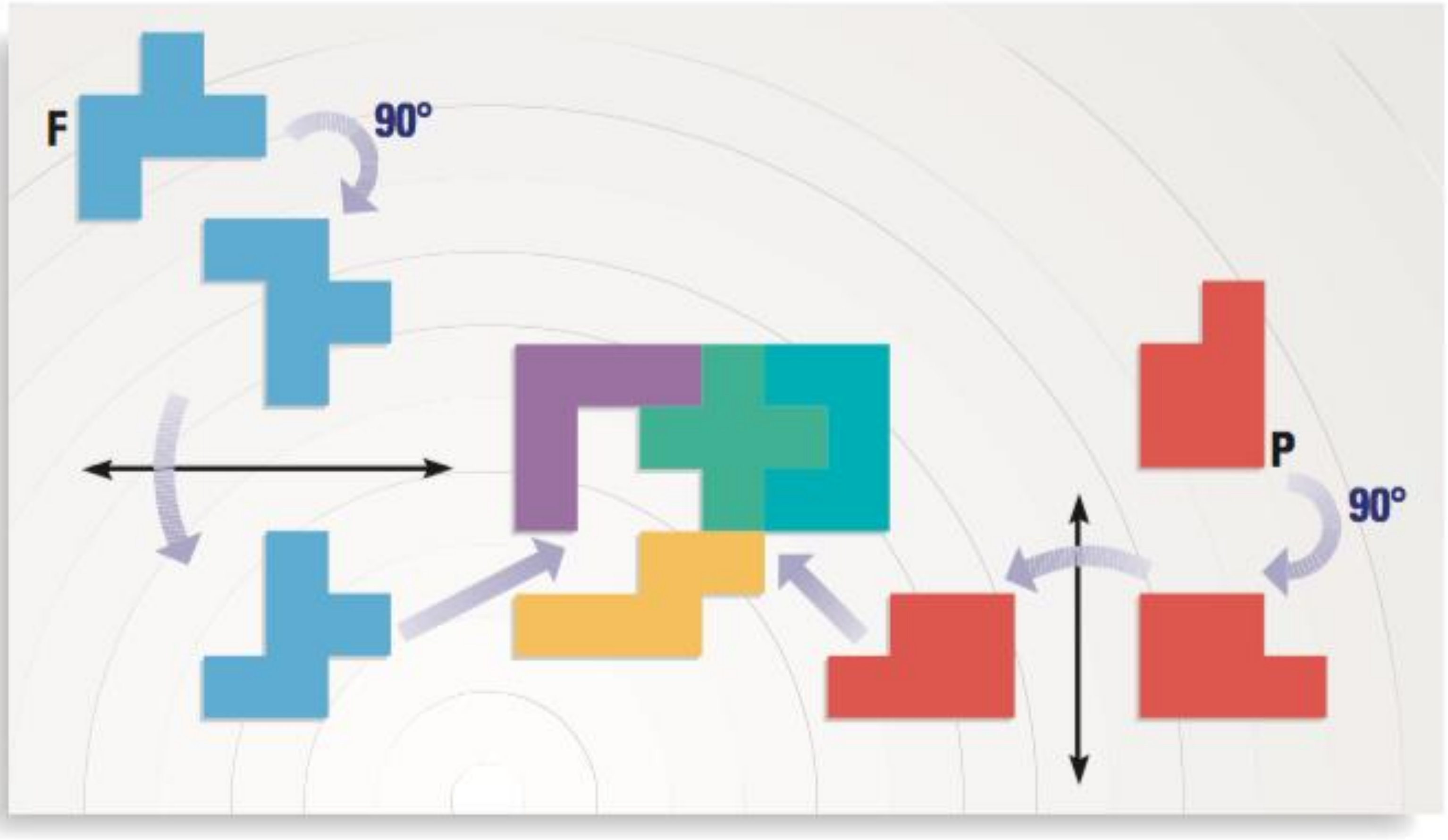
Example 5: Describing a Composition

PUZZLES The mathematical game pentominoes is a tiling game that uses twelve different types of tiles, each composed of five squares. The tiles are referred to by the letters they resemble. The object of the game is to pick up and arrange the tiles to create a given shape. Use compositions of transformations to describe how the tiles below will complete the 6×5 rectangle.

SOLUTION

To complete part of the rectangle, rotate the F tile 90° clockwise, reflect the tile over a horizontal line, and translate it into place.

To complete the rest of the rectangle, rotate the P tile 90° clockwise, reflect the tile over a vertical line, and translate it into place.



EXIT SLIP